The Pendulum and Standards of Measure in the Ancient World

Roland A. Boucher

When the French proposed their first metric system in 1723, they had no idea it had been invented by the ancient Mesopotamians 5000 years earlier. Just as the French proposed to use the length of a one-second pendulum to create standards of length, volume and weight, the Sumerians had created nearly identical meters, liters and kilograms. Our research shows that the Sumerians in ancient Mesopotamia used both the Moon and the Sun as their clock. It appears that the Egyptians improved on the timing accuracy by using the stars. Later the Minoans introduced the use of the planet Venus as a clock.

These concepts spread throughout the Ancient world from Britain in the West to Japan in the East. The Minoan standards are immortalized in the Magna Carta of 1215. The old English saying "a pint a pound the world around" had been true for over 3000 years. In the 19th Century, both Stuart and Penrose accurately measured the dimensions of the Parthenon finding its width to be 0.9997 arc seconds on the polar circumference of the Earth. This accuracy puzzled scholars for 150 years. Our research shows the width of the Parthenon in Athens was designed to be 1/30 of the perimeter of the Great Pyramid of Giza. The same pendulum formula, when timed with Venus rather than the Sun, increased the pendulum length just the right amount. This precision was not dumbfounding – it was just dumb luck.

Introduction

We intend to prove that ancient societies in the Third Millennium B.C.E. made use of the properties of the pendulum to develop precise standards of length, volume and weight and that these standards spread throughout the Ancient World, some surviving to modern times.

Our search began with the development of a number of proposed pendulums, each based on the timed motion of celestial objects such as the Moon, the Sun, the Stars, and the planet Venus.

Standards of volume and weight were then developed from the resulting pendulum lengths.

Next, these pendulum standards were compared with examples of standards of length, volume and weight which have survived for 4 or 5 millennia. Few physical standards of length and volume have survived, however many examples of ancient calibrated weights were found. This search for these standards took place over a number of years, in libraries and in the archives of archeological journals.

The Sumerians of ancient Mesopotamia used a sexagesimal numbering system, counting in multiples of both 6 and 10. The division of the meter or yard, called a Step, was divided into 60 Fingers as well into Cubits of 30 Fingers and Feet of 20 Fingers. The standard volume of approximately one liter was that of a 1/10 step or 6 finger wide cube called a Sila. The standard weight of approximately 1/2 kilogram, called the Mina, was 1/2 the weight of one Sila of water. The weight of 60 mina was called a talent and the weight of 1/60 mina was called a Shekel.

Many of these Ancient civilizations also retained a binary division of volume where the dimensions of the container are reduced in halves. As an example, the volume of a cubic foot, sometimes called an Amphora, was divided into 1/2 foot cubes creating an ancient gallon of 1/8 cubic foot, which was further divided into pints of 1/64 cubic foot. The binary standard of weight, the pound, was established as the weight of one pint of water. The binary division of the foot into 16 fingers was also common.

The Accuracy We Could Expect From an Ancient Pendulum

The Platinum ball and iron wire used by the French in their first proposed metric system would not have been available to the Sumerians. However, they would have

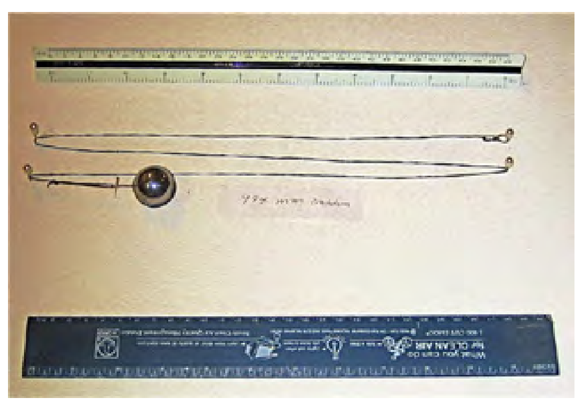

Figure 1. 994 mm Test Pendulum

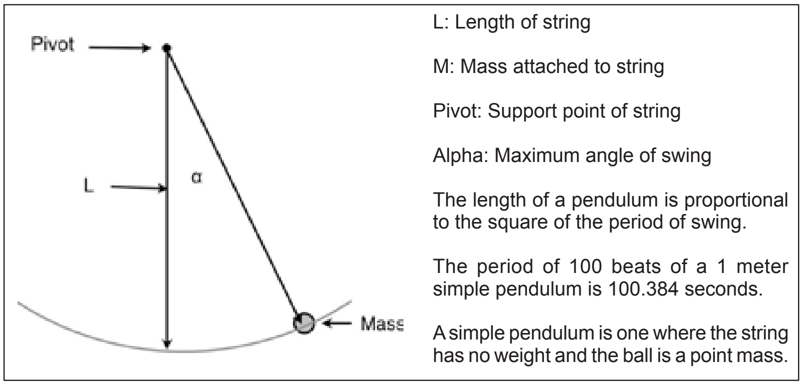

Figure 2. A Simple Pendulum

gold, copper, or stone available for the ball and waxed flax string to replace the iron wire. I constructed a number of such pendulums using brass or steel balls and waxed flax string and found that a 944 mm pendulum would consistently swing through 100 beats in 100.00 seconds, a precision of one part in 10,000. *The Sumerians could easily reproduce this pendulum.*

How a Pendulum Can Be Used to Establish a Unit of Length

Developing a comprehensive and precise set of measurement standards was not a trivial problem faced by ancient civilizations. Human body parts are not precisely reproduced from parent to child and are not suitable as standards of measurement. However, the names of some body parts have provided convenient names for some standards. Nature provides no standard of length, weight, or volume. Fortunately, it can provide four standards of time:

- the Diameter of the Full Moon passing by a line of sight can divide the day into 720 parts;
- the line of sight to the Sun rotating one degree in azimuth divides the solar day into 360 parts;
- the line of sight to a star rotating one degree in azimuth divides the day into over 360 parts; and
- the line of sight to Venus rotating one degree in azimuth divides the day into under 360 parts.

The "beat" of a pendulum is the time it takes the pendulum to make one-half swing, i.e., the time from when the swinging pendulum is vertical, or the point where it would hang if at rest, through the extension of the swing to its highest point, until it swings back and reaches vertical again.

The beat is determined almost exclusively by the Earth's gravity and the length from the pendulum's pivot point to the center of its mass. Unless the physical properties of a standard pendulum are carefully chosen, errors can occur [1, 2]. Increasing the angle of the swing will cause the period of a pendulum to increase about 600 parts per million when the swing increases to 5 degrees each side. Fortunately, the non-Zero weight of the string in a real standard pendulum will cause the period to decrease. This compensating effect in this case will completely cancel the increased period when the weight of the string reaches about 1/120 the weight of the ball. With care, five figure accuracy can be achieved. In any case, it would be highly unlikely to expect an error greater than 0.1 percent even if the user had no knowledge of this effect.

The force of gravity is relatively constant anywhere on the Earth's surface [3]. Therefore, the only two variables in a swinging pendulum are the length of the string and the time of the beat. If one of these two variables is known, the other can be determined. In other words, if two pendulums have the same length of string, they will have exactly the same beat anywhere on Earth. Conversely, if two pendulums have the same beat, then they will have exactly the same length of string. This fact, combined with the relative ease of building a pendulum, makes it an ideal and logical starting point for a universal system of linear measurement. The only prerequisite is determining an accurate measure of a time.

Establishing Accurate Intervals of Time

The simplest method which seems to have been used was to mark the interval of time it took the diameter of a full Moon to rise or set, or the interval of time it took the moon's disk to pass a north-south line of sight. This interval, when viewing the full moon near apogee is about 121 seconds. The length of a simple pendulum, allowed to complete 60 swings or 120 beats in this interval, would be about 1012 mm.

The second method which seems to have been used was to apply the interval of time it took the Sun's shadow to rotate through one degree west to east. This angle can be constructed by using a wheel to mark a distance of 180 diameters in a north-south direction from a peg. Then roll the wheel for 1/2 circumference to each side of the south end of the line, planting stakes at both ends, and stretching a string from the peg to each stake. The interval of time it takes for the center of the sun's shadow to move from one string to the next is 240 seconds. This interval is the Sumerian Gesh or 1/360 parts of the Solar day. The length of a simple pendulum, allowed to complete 120 swings or 240 beats in this interval, would be about 994 mm.

Element	Solar day	Star day	Venus day
length of day	***86400 sec***	***86164.08 sec***	86560.33 sec
length of 1/360 day	***240.00 sec***	***239.3447 sec***	240.4454 sec
length of 1/366 day	***236.065 sec***	***235.421 sec***	236.504 sec

Table 1. Timing intervals for Sun, Star, and Venus.

The Egyptians may have been the first to use a star to mark an interval of time. The interval of time it takes for a star in the equatorial plane to move 1/366 of a complete circle is 235.421 seconds. A star, as a mere pinpoint of light, can provide a much higher level of precision than the Sun. The length of a simple pendulum allowed to complete 366 beats in this interval would be about 300 mm.

The Minoans on the island of Crete may have been the first to use the planet Venus to mark the interval of time. The planet Venus is closer to the Sun than the Earth and orbits the sun in 244 days. By viewing Venus when it is on the opposite side of the Sun from the Earth, its motion cancels out some of the apparent motion caused by the spinning Earth. The interval of time it takes for Venus to move 1/366 of a circle is 236.504 seconds. The length of a simple pendulum allowed to complete 366 beats in this interval would be about 303 mm.

Creating an accurate interval of time of about four minutes or multiples thereof was obviously something the Ancients would have no trouble achieving.

Establishing Proof That a Pendulum Length is Actually an Ancient Standard of Length

While our study began in Sumeria, it soon took us to Egypt and to the Minoan civilization on the island of Crete. These studies convinced us that these three great civilizations of the ancient Near East had indeed used a pendulum to create their standards of length. Evidence was found that pendulum-derived standards of length were used in China and Japan to the East, Greece, Italy, Germany and France, and finally in England to the west. One of these ancient standards, the English pound, is still in use in the USA today. We suspect that the Ancient Japanese Foot may still be used by some Japanese carpenters and wood workers today.

Proof that these Ancient Standards of length were pendulum-derived is not always as easy as comparing pendulum-derived length to an Ancient standard of length. In some cases, the only standards that remain are those of volume or weight. Fortunately, standards of volume were derived directly from the cube of a linear dimension, just as is done today. Standards of weight, in turn, were derived from the weight of a standard volume of water at room temperature. (The metric system today specifies that the weight be established with the water temperature at 4 °C). Finally, some standards of weight were derived from the weight of a volume of grain; in some cases standards existed for a variety of grains in the same culture. Fortunately, a liquid standard using water was always established.

Creating Four Standards of Length Using a Pendulum in Ancient Sumeria

We cannot be sure of the chronological sequence of these developments, however they occurred in the third millennium BCE or earlier. The first three we will discuss using only the sexagesimal number 360 and its multiples; the last introduced the number 366 which represents the number of days in a star or sidereal year.

The Sumerian Pendulum of the Moon and Gudea

Pendulum 1 was timed with the Moon. Its length as a simple pendulum was 1012 mm. The proof of its existence is found in both a preserved standard of length and a preserved standard of volume as follows.

Berriman [4] states that Gudea was the Governor of Lagash circa 2175 B.C.E. In 1881, de Sarzec found eight headless statues of Gudea in the ruins of Lagash, a port city in Sumeria. Two of the statues show Gudea with a ruler on his lap. The ruler had a scale of 16 nominally equal divisions with a total length of 269 mm. The average

Figure 3. Gudea's Rule

Figure 4. Entemena's Vase

length of the divisions is 16.81 mm. If this length were a Sumerian Shusi then the length of Gudea's Sumerian Step of 60 Shusi (two cubits) would be 1008.75 mm. This length is only 0.4 percent shorter than our simple pendulum and is well within the expected range of a real pendulum. This would also establish Gudea's Foot at 1/3 Step, 20 Shusi, or 336.25 mm.

Berriman [5] writes that Entemena's Vase, a fine example of the silver smith's art (2400 B.C.E.) was found by de Sarzec during his excavation of ancient Lagash, at Tello; it is now in the Louvre. An inscription records its dedication by Entemena to the god Ningirsu in his temple of Eninnu, during Dudu's high priesthood; Entemena was the fifth governor at Lagash, during the Third Dynasty of Kish. Thureau-Dangin [5] published its volume as 4.71 liters. This is the volume of a 16.7626 mm cube. If this volume were 1000 cubic Shusi, or the volume of a 10 Shusi cube, the length of the 60 Shusi Seed Cubit would be 1005.75 mm. The volume of Entemena's Vase might also be considered a gallon of 1/8 a cubic foot. The corresponding foot would be 335.25 mm.

It appears that Gudea's Foot traveled to Europe where it became the Anglo Saxon Foot of 335.28 mm. This four-figure match in dimension is unlikely to have been the result of chance. This Anglo Saxon Foot then traveled to England where the Furlong of 600 Anglo Saxon Feet was used to establish all land boundaries. This Furlong later became the British Furlong of 660 British Imperial Feet from which all British linear measures were derived [6].

Detail of Calculations for a Lunar Pendulum

The length of the Lunar Pendulum 1 would be adjusted to swing through 120 beats in 121 seconds. In Table 2-A we establish the theoretical length for this simple pendulum, then applying modest corrections for the length of a real pendulum, we show the resulting volume of one Sila and the weight of one Mina. Finally, we show corresponding measured values from documented sources including a surprising match to the Anglo-Saxon Foot. In Table 2-B we establish the Bushel as a cubic foot, and in Table 2-C we establish the Talent of 60 Mina. We show these three variations because each has historical significance in the measured records.

Pendulum 1	Length,mm	Sila, ml	Mina, gm	Matching values
P=1.00833 sec	1008.95	1039.5	518.2	values for simple pendulum
L - 0.15%	1008.8	1030.82	515.4	1008.8 mm Gudea's Rule [4]
L + 0.26%	1011.6	1035.15	516.05	516 g # 16 Susa 5 shekel [7]
L + 0.26%	1011.6	1035.15	516.05	516 g # 15 Susa 2 shekel [7]
L -0.32%	1005.75	1017.35	507.37	506.6 g #20 Telloh 3 mina [7]
L - 0.32%	335.25	NA	NA	335.28 mm Anglo-Saxon Foot

Table 2-A. The length of the lunar pendulum with associated volume and weights standards.
Note: The Mina is 1/2 the weight of one Sila of water at room temperature.

Pendulum 1	R1	Length	Volume	R2	Weight	Matching Values
Bushel	20	335.25	37680	8000	37568	values for one cubic foot
Gallon	10	168.83	4710	1000	4696.03	4710 ml Entemena's Vase
Pint	5	84.417	588.7	125	587.0	no match
cubic finger	1	16.883	4.71	1	4.696	1/1000 Entemena's Vase

Table 2-B. The Cubic Foot as a Bushel with Gallon, Pint, and Cubic Finger (L -0.32 percent).
Note: R1 is the length of the equivalent cube in fingers R2 is the volume in cubic fingers. The weights are computed from the volume of water at room temperature.

Pendulum 1	Ratio	Weight gm	Matching Values	
Talent	60	30903	30900 g	Talent # 13 Arthur Evans, Crete [8]
Mina	1	516.05	516 g	#5 and #2 Susa shekels [7]
Shekel	1/60	8.6	4.29 x 2 g	# 17 half shekel [7]

Table 2-C. The Mina, Talent of 60 Mina, and Shekel of 1/60 Mina (period adjusted +0.26 percent).


Figure 5. Mina N


Figure 6. Limestone Duck

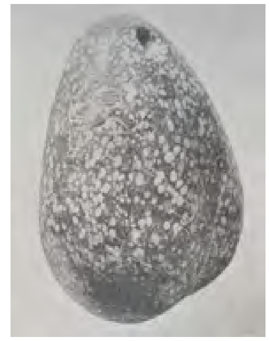
Figure 7. Mina D

Pendulum 2: The One Second Pendulum

Pendulum 2 was timed with the Sun and was allowed to beat 240 times in 1/360 Solar day or 240 seconds. The length of this simple one-second pendulum at the latitude of Lagash is 992.34 mm. The proof of its existence is found preserved in a number of ancient standards of weight.

In the British Museum, there is a weight (No. 91005) that Berriman calls Mina N because its inscription certifies it to be a copy of a weight that Nebuchadnezzar II (605-562 B.C.E.) made matching a weight that belonged to Shulgi of the Third Dynasty of Ur (c 2100 B.C.E.) [9]. Its mass weighed by Belaiew was 978.3 gm. This weight is equal to 981.1 ml of water at room temperature (20 °C)—the volume of a Sila created from a 993.7 mm Sumerian Step or Double Cubit. The length of a physical one-second pendulum with ball/string ratio of 60 and a half swing of 1/10 its length would be is 993.7 mm. The weight of a Mina derived from this Sila is 489.2 grams and identical to the Mina N.

In the Ashmolean Museum, there is Babylonian Limestone Duck weight from Erech [10]. Its published mass is 2417 gm. If intended to be 5 mina in mass, one mina would equal 483.4 gm. A Talent of 60 mina would weigh 29,004 grams. Its volume of water at room temperature of 20 °C would be 29090 ml. This is the volume of a 307.55mm cube. A double cubit at 3 times this length would be 992.65 mm, a little longer than our simple pendulum. We conclude that the weight of the Babylonian Limestone Duck Weight is derived from our simple one-second pendulum.

Mina D is the oldest extant weight in the Ashmolean Museum at Oxford England. It was signed by Dudu, the high priest at Lagash, c 2400 B.C.E. Berriman reports that it was measured at 680.485 grams [11]. This weight is exactly 150 Sumerian and 100 Minoan gold standards, as well as 50 Egyptian Old Kingdom Deben. Mina D and seven other gold standards are exact multiples of the weight of one cubic finger of water.

When the French proposed their first metric system in the 18th century, they were unaware that it was already over 5000 years old and memorialized in the Mina N. The original French proposal for a metric system in the early eighteenth century defined the meter as the length of a one-second pendulum (993.7 mm) when measured in the Earth's gravitational field at 45 degrees north latitude. Rounding off the length to 994 mm, we maintain excellent correlation to the French Pendulum; the length of the Sumerian double cubit becomes 1.003 original French meter. The volume of the Sumerian Sila (liter) and the weight of the Sumerian Double Mina become 1.009 original French liters and kilograms.

Detail of Calculations for the One Second Pendulum

Pendulum 2 beat 240 times in 240 seconds. In Table 3-A, we establish the theoretical length for this simple pendulum, then applying modest corrections for the period and length of a real pendulum, we show the resulting Sila and Mina. Finally, we show corresponding measured values from Powell and Berriman. In Table 2-B, we establish the foot, the bushel as a cubic foot, and its division into gallon, pint, and Cubic Finger. Finally, we show one corresponding measured value. In Table 2-C, we establish the Talent of 60 Mina and shekel of 1/60 Mina showing corresponding measured values. We show these three variations because each has historical significance.

Alternative lengths for the Sumerian Foot of Pendulum 2 seem to have been established where 1000 feet rather than 1080 feet were equal to the length of the Cable of 360 Step Cubits or pendulum lengths. The length of the simple one-second pendulum in Lagash was 992.34 mm. The length of this new foot would be (360/1000) x 992.376 = 357.255 mm. A pendulum of this length could be obtained directly using 400 beats in 1 Gesh of 240 seconds. This pendulum would be too short to time easily, but one four feet long would work well. It would beat 200 times in 240 seconds. We found no matches in the signed weights of

Pendulum 2	Length, sec	Sila, ml	Mina, gm	Matching Values
P = 1.0000	992.340	977.310	487.210	487.2 g #52 5 Shekels [7]
+ 650 ppm	993.670	981.130	489.120	993.7 mm French Meter
+ 650 ppm	993.670	981.130	489.120	978.3 g #50 Mina N [7]
- 1300 ppm	989.800	969.720	483.400	483.4 g Limestone Duck [10]
- 400 ppm	991.590	975.000	486.050	486.3 g #53 30 mina [7]
- 400 ppm	991.590	975.000	486.050	486 g #54 5 Shekels [7]

Table 3-A. The length of Pendulum 2 and associated volume and weight standards.

Pendulum 2	R	Length	Volume	R	Weight	Matching Values
Bushel	20	331.223	36338	8000	36231	no match
Gallon	10	165.612	4542.3	1000	4528.88	no match
Pint	5	82.806	567.78	125	566.105	no match
cu finger	1	16.561	4.542	1	4.5288	4.53656 = #2 Mina D [7]

Table 3-B. The Cubic Foot as a Bushel divided into Gallons and Pints (period + 650 ppm).
Note: The weight of this cubic finger of water was the Sumerian Gold Standard.
Note: R2 is the volume in cubic fingers; R1 is the length of the equivalent cube in fingers.

Pendulum 2	R	Weight	Matching values	
Talent	60	29347	29,400	Arthur Evans Talent At Knossos [8]
Mina	1	489.1	489.154	Mina N [7]
Shekel	1/60	8.08	7.95	#65 (5.3) Samas 2/3 Shekel [7]

Table 3-C. The Talent of 60 Mina with corresponding Shekel (period + 650 ppm).

Pendulum 2	Length mm	Sila ml	Mina gm	Matching Values
P = 1.0000	992.34	977.31	487.21	487.2 g see Table 2-A
+ 650 ppm	993.67	981.43	489.30	993.7 mm French meter 1793
Foot @ 0.36 L	357.72	NA	NA	360mm Zhou Royal Ch ih [13]
Foot @ 0.36 L	357.72	NA	NA	357.2 mm Bordeaux France [14]

Table 4-A. Alternate One-second Pendulum with 360 pendulum lengths = 1000 Feet.

Pendulum 2	R	Length mm	Volume ml	R	Weight gm	Matching Values
Bushel	1	357.8	45806	1	45671	no match
Gallon	1/2	178.9	5725.7	1/8	5708.8	no match
Pint	1/4	89.45	715.72	1/64	713.6	no match

Table 4-B. The Cubic Foot as a Bushel divided into Gallons and Pints (period +650 ppm).

Pendulum 3	Length mm	Sila ml	Mina gm	Matching Values
P = 0.66667	882.11	686.39	342.18	no match
Foot @ 0.36L	317.56	NA	NA	318 mm China Zhou Market Foot [13]
Foot @ 0.36L	317.56	NA	NA	317 mm fuss Bern Austria [15]

Table 5-A. Lengths (mm), Sila (cu cm), Mina (grams) with some Matching Values.

Pendulum 3	Length mm	Volume ml	R	Weight gm	Matching Values
Bushel	317.56	32,024	1	31,929	32,000 g Talent # 14 A.E. Evans
Gallon	158.78	4003	1/8	3991	no match
Pint	79.39	500.4	1/64	498.9	498.67 g # 31 Shulgi 10 Minas [[7]
Pint	79.39	500.4	1/64	498.9	498.468 g #32 Telloh 5 Shekels [7]
Pint	79.39	500.4	1/64	498.9	498 g #34 2 Minas [7]
1/60 talent	NA	NA	NA	532.15	534.2 g #12 1/6 Mina [7]

Table 5-B. The Cubic Foot as as Bushel, Gallon, and Pint along with associated weights.

Pendulum 4 period	Length, mm	Sila, ml	Mina, gm	Measured values
P=0.99727	986.930	961.300	479.229	479.6 # 61, #58 5 Shekels [7]
Foot @ 1/3 L	328.997	NA	NA	329 mm Assyrian Foot [12]
+ 650 ppm	988.213	965.054	481.101	481.07 #56 1/2 Mina [7]
+ 650 ppm	988.213	965.054	481.101	480.145 #57 1 Mina [7]
- 650 ppm	985.649	957.561	477.365	477.28 # 63 2 Mina [7]
- 1000 ppm	984.959	955.553	476.364	476.1 # 66 no name [7]

Table 6-A. Timing the One Second Pendulum with a Star Created the Assyrian Foot.

Pendulum 4	L	Length mm	Volume ml	R	Weight gm	Measured values
Bushel	1	329.405	35,743	1	35,637	8000 cubic fingers
Gallon	1/2	164.702	4,467.8	1/8	4,455.00	1000 cubic fingers
Pint	1/4	82.351	558.48	1/64	557	557.81 #7 1/6 mina [7]
Mina	NA	NA	595.72	1/60	594	no match
Mina (grain)	NA	NA	595.72	1/60	475.2	475 g # 67 3 mina [7]

Table 6-B. A Bushel of one Cubic Foot divided into Gallon, Pint, and Mina (period + 650 ppm).

Dr. Powell, but did find two good matches in France and China. This is the length of Royal Ch'Ih of the Chinese Zhou Dynasty at 360 mm [13] and the "Pied de Terre" of Bordeaux France at 357.24 mm [14].

Detailed calculations to establish the bushel from our new cubic foot derived from Pendulum 2, as well as its division into Gallons and Pints, showed no match as shown in Table 4-B.

Pendulum 3 and the Chinese Market Foot

Pendulum 3 beat 360 times in 240 seconds. This Pendulum was quite short, so double its length was used. A Cable of 360 double pendulum lengths was equal to 1000 Feet. This new standard Foot seems to have been adopted in later cultures. Proof of its existence can be found in lengths described by A.E. Berriman and in weights described by both M.A. Powell and Sir Arthur Evans. Its double length in Lagash was 882.08 mm with the length of the new foot 317.55 mm. This is the length of the Market Foot in the Chinese Zhou Dynasty, at 318 mm [13] and the Steinbrecherfuss of Bern, Austria at 317 mm [15].

Detailed calculations to establish the volume of the Bushel, Gallon, and Pint from this new Cubic Foot derived from Pendulum 3, as well as the weight of water at room temperature associated with each, as shown in Table 5-B.

Pendulum 4: The Assyrian Foot of Babylon

The Egyptian method of timing a pendulum with a star was later adopted by the Assyrians in Babylon c 1750 B.C.E. The original Sumerian one-second pendulum was allowed to swing the same 240 beats, but in 239.3447 seconds or 1/360 of a celestial day. This pendulum appears to have created the Assyrian Foot and provided a match to six signed weights in references 7 and 12.

Conclusion

Chapter One established five Ancient Sumerian Standards of length through 32 matches among Dr. Powell's inscribed weights, 3 matches among Sir Arthur Evans' Talent weights, and among 7 of Mr. Berriman's lengths, volumes, and weights.

Please join us again in following articles of *CAL LAB* magazine, where we will examine additional standards from Egypt and the Minoan civilization on Crete. Finally, we will establish the design perimeter of the Great Pyramid of Giza at 30 arc seconds and explain the luck of the Greeks who used a version of this formula to establish the width of the Parthenon at almost exactly one arc second on the polar circumference of the Earth.

References

[1] Arthur Bronwell, *Advanced Mathematics in Physics and Engineering*, (New York: McGraw-Hill. 1953), 137-139.

[2] E. Janhke, and F. Emde, *Table of Functions*, (New York: Dover Publications Fourth Edition 1945), 85.

[3] Glen Thorncroft, *Accuracy of Gravity*, ME department California Polytechnic State University, http://www.calpoly.edu/~gthorncr/ME302/documents/AccuracyofGravity.pdf.

[4] A.E. Berriman, *Historical Metrology*, (E.P.Dutton & Co, 1953), 53-54.

[5] Berriman, 63-64.

[6] Ronald E. Zupko, *British Weights and Measures*, The University of Wisconsin Press, 20.

[7] M.A. Powell, *Sumerian Numeration and Metrology*, University of Michigan, 1973, 205.

[8] Arthur J. Evans, "Minoan weights and mediums of currency from Crete, Mycenae, and Cyprus Corolla Numisimatica," *Numismatic Essays in Honor of Barclay V. Head*, 342.

[9] Berriman, 58.

[10] Berriman, 62.

[11] Berriman, 56.

[12] Berriman, 55.

[13] Berriman, 47.

[14] P. Guilhiermoz, *A propos d'une publication récente Bibliothèque de l'Ecole des Chartes, Del'équivalence des anciennes mesures*, 1913, volume 74, 27.

[15] "Obsolete German units of measurement, " https://en.wikipedia.org/wiki/Obsolete_German_units_of_measurement.

Roland A. Boucher (rolandfly@sbcglobal.net), Irvine, California.